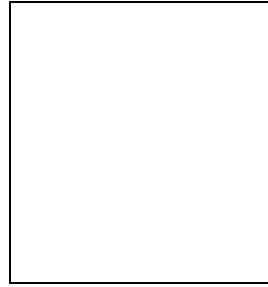


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$K \rightarrow \pi\nu\bar{\nu}$ AND R-PARITY VIOLATING SUPERSYMMETRY

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Motivated by the recent experimental data of the E787, E949 and E865 collaborations and by the difference between the standard model (*SM*) prediction and data, we consider in detail R-parity violating (*RPV*) supersymmetric contributions to $K \rightarrow \pi\nu\bar{\nu}$. The theoretical cleanliness of this decay constitutes a useful way to provide constraints, independent of long distance effects. Including the possibility of interferences between one-loop *R*-parity conserving (*RPC*) supersymmetry and tree-level *RPV* supersymmetric contributions, our results allow to improve the limits on *R*-parity violating couplings with respect to previous analyses.

1 $K \rightarrow \pi\nu\bar{\nu}$ in the standard model

The process $K^+ \rightarrow \pi^+\nu\bar{\nu}$ is governed in the *SM* by the following effective Hamiltonian¹

$$H_{eff} = \frac{G_f}{\sqrt{2}} \frac{2\alpha_e}{\pi \sin^2 \theta_w} \sum_l \left(\lambda_c X_c^l + \lambda_t X_t \right) s_L^- \gamma^\mu d_L \bar{\nu}_L^l \gamma_\mu \nu_L^l + h.c., \quad (1)$$

where $\lambda_i = V_{is}^* V_{id}$ are products of CKM² matrix elements. The loop-function X_t contains the top contribution, and X_c^l the charm contribution for flavour l . In the computation of the branching ratio, the hadronic matrix element can be related via isospin to the experimentally well known decay $K^+ \rightarrow \pi^0 e^+ \nu_e$ ³. It's branching ratio has recently been measured with high statistics by the E865 collaboration⁴. However, their result, $BR(K^+ \rightarrow \pi^0 e^+ \nu_e) = (5.13 \pm 0.15) \times 10^{-2}$, differs considerably from the most recent value of the Particle Data Group⁵, $(4.87 \pm 0.06) \times 10^{-2}$, which does not include yet the above mentioned result. So, we will use for our analysis an average value, where we take into account the Particle Data Group fit as well as the E865 result: $BR(K^+ \rightarrow \pi^0 e^+ \nu_e)_{av} = (5.08 \pm 0.13) \times 10^{-2}$. With updated values of CKM elements and quark

masses, our standard model prediction at one-loop for the branching ratio of $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ is :

$$BR(K^+ \rightarrow \pi^+ \nu \bar{\nu})^{SM} = (8.2 \pm 1.2) \times 10^{-11} \quad (2)$$

which is still compatible with the recent experimental result⁶, $(1.47^{+1.3}_{-0.8}) \times 10^{-10}$. However, the predicted central value is half the observed value so possible new physics effects should be of the same order as the *SM* ones in order to get the measured central value. In this paper, we will be concerned with obtaining limits on *R*-parity violating couplings of supersymmetric extensions of the *SM*.

2 R-parity conserving supersymmetry

At this stage we will assume unbroken *R*-parity. Then, just as in the *SM*, there are no supersymmetric contributions at tree level. They start only at one-loop order.

The determination of the supersymmetric contribution is obtained in the same way as in the *SM* case : with the effective Hamiltonian of Eq. 1 where X_t is now replaced by⁷ $X_{new} = r_K e^{-i\theta_K} X_t$. r_K and θ_K parameterize new physics contributions and are functions of masses and couplings of the new particles. The *SM* is then included as a special case, where $r_K = 1$ and $\theta_K = 0$. With a *MSSM*-like field content, the standard model particles, the charged higgses, the charginos and the neutralinos enter in the loops. Unfortunately the results for the branching ratio are very sensitive to the yet-unknown *SUSY* parameters (masses and couplings). However, it is possible to estimate the order of magnitude of the *R*-parity conserving (*RPC*) contributions. The authors of Ref.⁷ found for r_K and θ_K the typical ranges^a

$$0.5 < r_K < 1.3, \quad -25^\circ < \theta_K < 25^\circ \quad (3)$$

by varying all *SUSY* parameters within the bounds allowed by experimental constraints. Then, varying r_K and θ_K within these ranges makes at most a change of $\sim 50\%$ ^b for the branching ratio of $K^+ \rightarrow \pi^+ \nu \bar{\nu}$:

$$BR(K^+ \rightarrow \pi^+ \nu \bar{\nu})^{RPC \text{ } SUSY} \simeq (8.2^{+4.3}_{-5.2}) \times 10^{-11}. \quad (4)$$

So we see explicitly that contributions of *RPC* supersymmetry can be of the same order of magnitude as the standard model ones.

3 R-parity violating supersymmetry

By including *R*-parity violation (*RPV*) in a supersymmetric extension of the standard model, we have now to consider new terms in the superpotential which allow baryon and lepton numbers violation. These are of the form⁹ :

$$W_{RPV} = \lambda_{ijk} L_i L_j E_k + \lambda'_{ijk} L_i Q_j D_k^c + \lambda''_{ijk} U_i^c D_j^c D_k^c \quad (5)$$

RPV couplings λ'_{ijk} induce tree level contributions via squark exchanges to the decay $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ (cf the diagrams shown in Fig. 1), so only the second term will be considered here.

^aThese ranges only indicate the most probable values. Our updated analysis agrees with this statement slightly enhancing the probability.

^bHowever, some particular points outside the ranges (3) and some points of the parameter space of the general *MSSM*, where some assumptions made here have been relaxed⁸, can give larger branching ratio and can saturate the experimental central value.

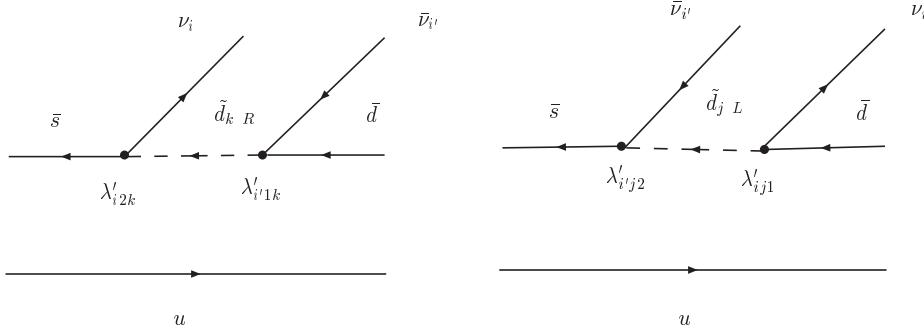


Figure 1: *R-parity violating tree level diagrams contributing to the process $K^+ \rightarrow \pi^+ \nu \bar{\nu}$.*

The branching ratio for the rare decay $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ can be written in the following form:

$$BR(K^+ \rightarrow \pi^+ \nu \bar{\nu}) \propto \left(\sum_l |C_l^{RPC} + \epsilon_{ll}|^2 + \sum_{k \neq l} |\epsilon_{kl}|^2 \right) \quad (6)$$

where C_l^{RPC} is the *R*-parity conserving contribution proportional to $(\lambda_c X_c^l + \lambda_t X_{new})$, the sum is over ν 's and anti- ν 's flavours, and the *R*-parity violating couplings are contained in the ϵ_{ij} :

$$\epsilon_{ij} = \sum_n \left(\frac{\lambda'^*_{i2n} \lambda'_{j1n}}{m_{\tilde{d}n_R}^2} - \frac{\lambda'^*_{in1} \lambda'_{jn2}}{m_{\tilde{d}n_L}^2} \right). \quad (7)$$

Under some assumption it is possible to constrain certain combinations of couplings, which will be done in the next section.

4 Constraints

From our previous discussion, we have drawn the conclusion that *RPC* supersymmetry has to be included in the analysis of $K^+ \rightarrow \pi^+ \nu \bar{\nu}$. Since we aim to obtain an upper-bound on the *RPV* contributions, we assume the *RPC* contributions (which already include *SM* ones) to be minimal (corresponding to $r_K = 0.5$ and $\theta_K = 25^\circ$) in order to allow for the largest possible contribution from *RPV* terms.

In contrast to the standard model and the *RPC* supersymmetric contributions, *R*-parity violating couplings can induce tree level processes with a neutrino and an antineutrino of different flavour in the final state. The *R*-parity violating processes with the same neutrino flavour in the final state, then, interfere with the *SM/RPC SUSY* contributions as can be seen from the first term in Eq. 6.

Just to see later the effect of the interferences, we neglect them in a first step. This leads to the bounds (setting all the couplings to zero except one product) :

$$|\frac{\lambda'^*_{i2n} \lambda'_{i1n}}{m_{\tilde{d}n_R}^2}|, |\frac{\lambda'^*_{in1} \lambda'_{in2}}{m_{\tilde{d}n_L}^2}| < \frac{2.1 \times 10^{-5}}{(200 \text{ GeV})^2} \quad (8)$$

But more realistic and precise constraints should take into account interferences. This, however, makes the extraction of upper bounds harder and no simple bounds can be given. In the following, we will assume that only final states with the same neutrino flavour occur. Thus, only ϵ_{ij} with $i = j$ has to be taken into account. The general equation verified by the ϵ_{ii} can be

written in the following way:

$$\sum_{i=e,\mu,\tau} \left(\text{Re}(\epsilon_{ii}) + \frac{\alpha_i}{2} \right)^2 + \sum_{i=e,\mu,\tau} \left(\text{Im}(\epsilon_{ii}) + \frac{\beta}{2} \right)^2 = R^2. \quad (9)$$

Taking only one of the ϵ_{ii} nonzero, this equation describes a circle in the complex plane, whose parameters can be found in the original paper¹⁰. As an example, the resulting constraints in the complex plane on ϵ_{11} are displayed in Fig. 2. To have a numerical idea of the interferences, we may choose the point of coordinates ($\text{Re}(\epsilon_{11})=-2$, $\text{Im}(\epsilon_{11})=-2$) on the “SUSY” circle of Fig. 2. It is approximately the point which gives the maximum value for $|\epsilon_{11}|$: $|\epsilon_{11}|_{max} \simeq 2.8 \times 10^{-5}$. That leads to :

$$| \frac{\lambda'_{i2n} \lambda'_{i1n}}{m_{\tilde{d}n_R}^2} |, | \frac{\lambda'_{in1} \lambda'_{in2}}{m_{\tilde{d}n_L}^2} | < \frac{2.8 \times 10^{-5}}{(200 \text{ GeV})^2} \quad (10)$$

Constraints on ϵ_{22} and ϵ_{33} can be obtained in the same way and are of the same order of magnitude, the limits (10) can be used for the 3 flavours.

These upper-bounds are 30% bigger than without interferences and so, our conclusion is that interferences do have a significant influence.

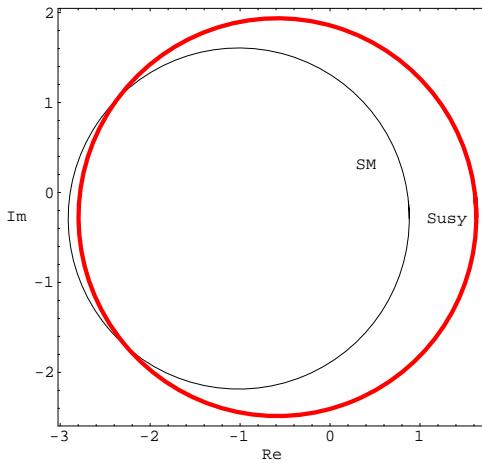


Figure 2: Allowed region for $\text{Re}(\epsilon_{11})$ and $\text{Im}(\epsilon_{11})$ in units of 10^{-5} , for the case of the standard model ($r_K = 1$ and $\theta_K = 0$, thin black circle) and for the “minimal” *RPC SUSY* ($r_K = 0.5$ and $\theta_K = 25^\circ$, red circle). The reference value for the mass of the squarks is 200 GeV.

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